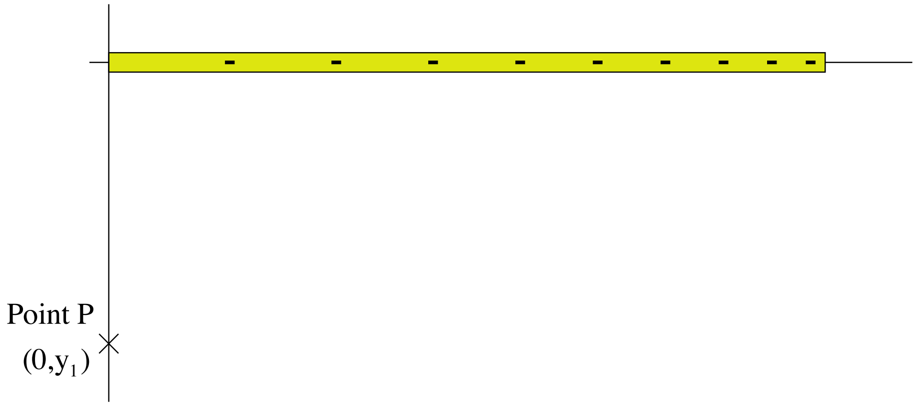
Negatively Charged, Insulating Beam with a Twist

A negatively charged rod of length "L" has a linear charge density of magnitude kx, where "k" is a constant and "x" a distance down the axis from the left end. Determine the electric field at Point P at

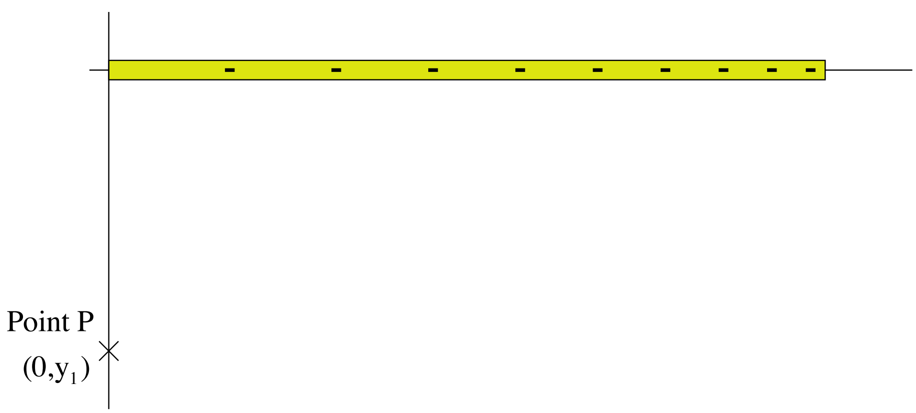


a.) What are the units of "k?"

b.) Determine the electric field at Point P.

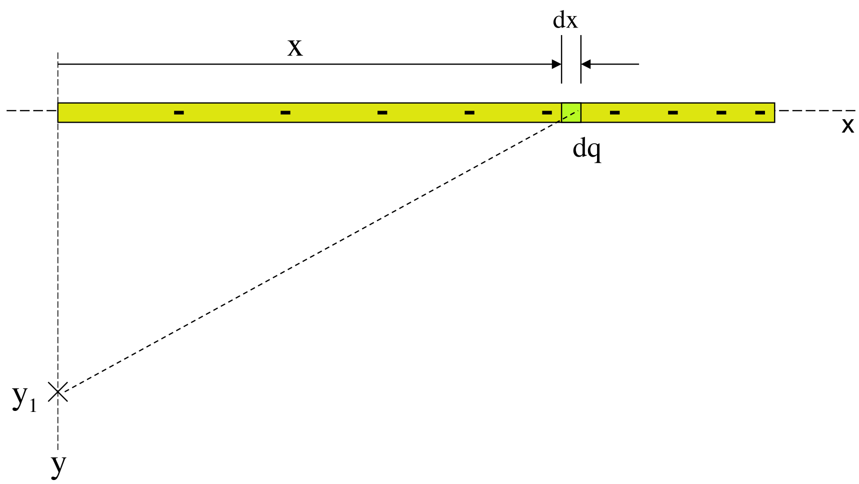
a.) What are the units of "k?" $\left[\lambda\left(\text{coul/m}\right)\right]=\left[k\left(\text{coul/m}^2\right)\right]\left[x\left(\text{m}\right)\right]$

b.) Determine the electric field at Point P.



The approach is always going to be the same:

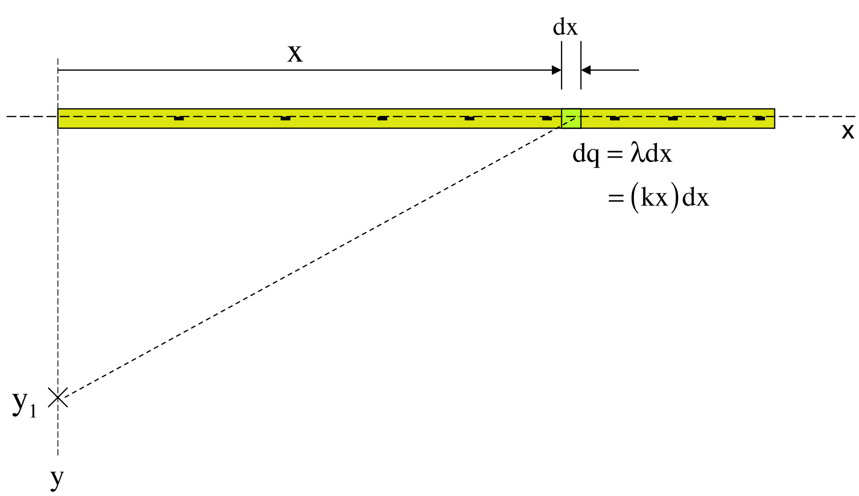
Identify a bit of differential charge dq over a differential length dx of section located at an ARBITRARY POINT x on the structure.



3.)

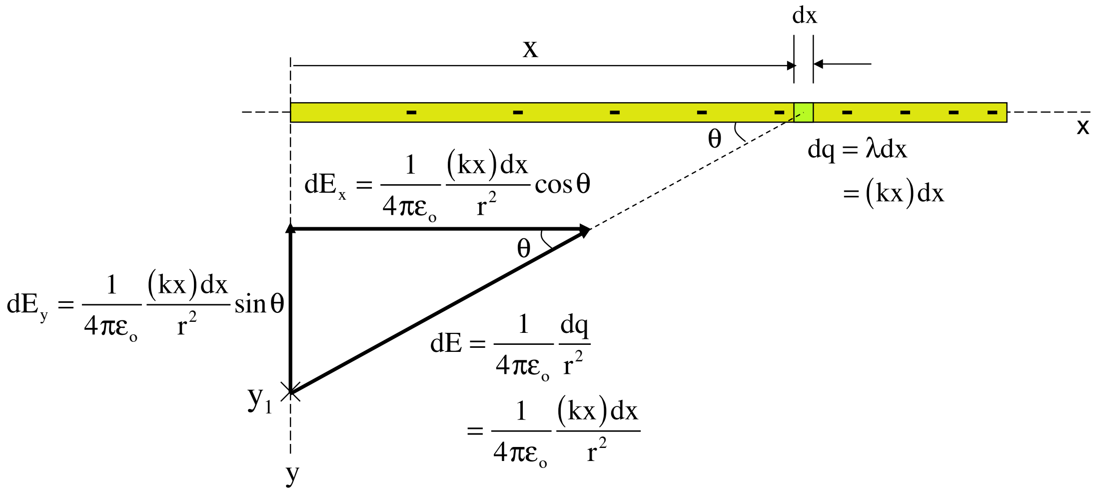
The approach is always going to be the same:

Express dq in terms of a linear charge density function (λ), an area charge density function (σ) or a volume charge density function (ρ).



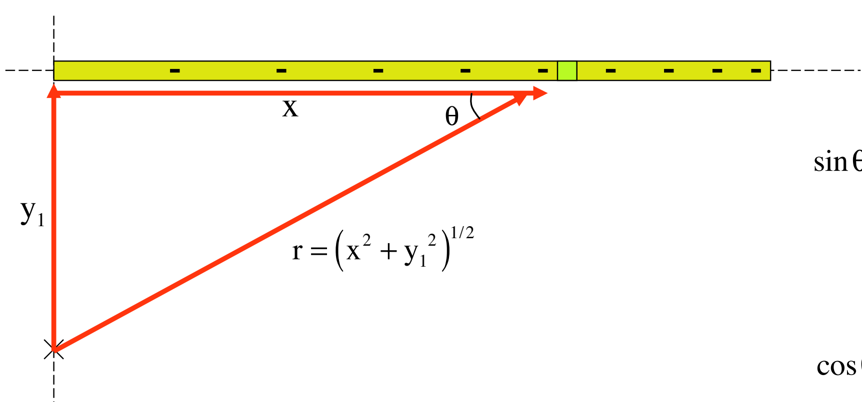
4.)

Determine the direction of the differential electric field dE at the point of interest due to dq and label it with an expression for dE’s magnitude. Once done, identify an angle within the system and break dE into its x-component and y-component.



5.)

There will be sine and cosine functions in the component expressions for dE. Determine these quantities in terms of the physical geometry of the system, noting that the sine of an angle is equal to the opposite side divided by the hypotenuse, and the cosine equals the adjacent side divided by the hypotenuse.



$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{y_1}{(x^2 + y_1^2)^{1/2}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{x}{(x^2 + y_1^2)^{1/2}}$$

Once done, put it all together and do the Calculus.

6.)

y – direction

$$E_y = \int dE_y = \frac{1}{4\pi\varepsilon_o}\int \frac{(kx)dx}{r^2}\sin\theta$$

$$= \frac{1}{4\pi\varepsilon_o}\int \frac{(kx)dx}{\left[\left(x^2+y_1^{\,2}\right)^{1/2}\right]^2}\left(\frac{y_1}{\left(x^2+y_1^{\,2}\right)^{1/2}}\right)$$

$$= \frac{1}{4\pi\varepsilon_o}\int \frac{(kx)dx}{\left[\left(x^2+y_1^{\,2}\right)^{1/2}\right]^2}\left(\frac{y_1}{\left(x^2+y_1^{\,2}\right)^{1/2}}\right)$$

$$= \frac{ky_1}{4\pi\varepsilon_o}\int_{x=0}^{L} \frac{x}{\left(x^2+y_1^{\,2}\right)^{3/2}}\,dx$$

$$= \frac{ky_1}{4\pi\varepsilon_o}\left.\frac{-1}{\left(x^2+y_1^{\,2}\right)^{1/2}}\right|_{x=0}^{x=L}$$

$$= \frac{ky_1}{4\pi\varepsilon_o}\left[\left(\frac{-1}{\left(0+y_1^{\,2}\right)^{1/2}}\right)-\left(\frac{-1}{\left(L^2+y_1^{\,2}\right)^{1/2}}\right)\right]$$

$$= \frac{k}{4\pi\varepsilon_o}\left[\left(\frac{y_1}{\left(L^2+y_1^{\,2}\right)^{1/2}}\right)-1\right]$$

x – direction

$$E_x = \int dE_x = \frac{1}{4\pi\varepsilon_o}\int \frac{(kx)dx}{r^2}c0s\theta$$

$$= \frac{1}{4\pi\varepsilon_o}\int \frac{(kx)dx}{\left[\left(x^2+y_1^{\,2}\right)^{1/2}\right]^2}\left(\frac{x}{\left(x^2+y_1^{\,2}\right)^{1/2}}\right)$$

$$= \frac{1}{4\pi\varepsilon_o}\int \frac{(kx)dx}{\left[\left(x^2+y_1^{\,2}\right)^{1/2}\right]^2}\left(\frac{x}{\left(x^2+y_1^{\,2}\right)^{1/2}}\right)$$

$$= \frac{ky_1}{4\pi\varepsilon_o}\int_{x=0}^{L} \frac{x^2}{\left(x^2+y_1^{\,2}\right)^{3/2}}\,dx$$

Yikes: you'd probably stop here unless you were a Calculus wizard

$$= -\frac{ky_1}{4\pi\varepsilon_o}\left[-\frac{x}{\left(x^2+y_1^{\,2}\right)}+\log\left(x+\left(x^2+y_1^{\,2}\right)\right)\right]\Bigg|_{x=0}^{x=L}$$

$$= \frac{ky_1}{4\pi\varepsilon_o}\left[\left(-\frac{L}{\left(L^2+y_1^{\,2}\right)}+\log\left(L+\left(L^2+y_1^{\,2}\right)\right)\right)-\left(\log\left(y_1^{\,2}\right)\right)\right]$$

What's the point?

The point is that although the problem looks impossible at the start, if you just follow the procedure it will walk you through the evaluation.

a.) Identify a bit of differential charge dq over a differential length dx of section located at an ARBITRARY POINT x on the structure.

b.) Express dq in terms of a linear charge density function (λ), an area charge density function (σ) or a volume charge density function (ρ).

c.) Determine the direction of the differential electric field dE at the point of interest due to dq (in what direction would a positive test charge accelerate due to dq if put at the point of interest?) and label it with the expression for dE's magnitude.

d.) If you are lucky and there is symmetry you can exploit, do so.

e.) Once done, identify an angle within the system and break dE in its x-component and y-component using algebraic expressions for the sine and cosine quantites.

f.) Finally, use an integral (complete with limits) to sum all the x-component differential electric fields, and do the same with the y-components fields.